**Proceedings of the ASME/BATH 2019
Symposium on Fluid Power and Motion Control
FPMC2019
October 7-9, 2019, Sarasota, FL, USA**

FPMC2019-1668

A MORE ACCURATE DEFINITION OF MECHANICAL AND VOLUMETRIC EFFICIENCIES FOR DIGITAL DISPLACEMENT® PUMPS

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ABSTRACT

The apparent volumetric displacement of digital displacement pumps and motors is reduced with increasing fluid pressure. So-called pump shrinkage has been documented in previous publications, where fluid compressibility effects were assumed to affect input and output power equally. In this paper, the authors derive the torque and flow rate of an ideal digital displacement pump. It is shown that the output power shrinks slightly more than the input. The difference between input and output shrinkage is counted as a power loss according to the accepted definition of total efficiency. New equations are presented for calculating mechanical and volumetric efficiencies which are up to 2% more accurate than the previous method (which assumes equal shrinkage) and up to 5% more accurate than conventional equations (which assume no shrinkage). Compressibility effects may be even more significant depending on pump design parameters, fluid properties, flow control algorithms and operating conditions. Calculations of partial pump efficiencies require a derived displacement volume to be known. The derived displacement volume of digital displacement pumps is considered for the first time in this paper. The contributions of this work are instructive for understanding the unique characteristics of digital displacement pumps as well as check-valve type pumps in general.

INTRODUCTION

Background

For the past seventy-five years, the swash-plate type, variable displacement pump has been used in applications where variable pump flow is needed. However, limits have been reached concerning the control and efficiency of these machines [1,2]. Traditional hydromechanical control systems are giving way to electronic controls in pursuit of greater performance and efficiency. Today's electrohydraulic pump controls typically consist of solenoid coil actuators and sensors for measuring fluid pressure and swash plate position. Yet the basic pump design principles, along with the efficiency and dynamic response, remain unchanged. New digital fluid power technology embodies digital design principles within the fluid domain and a deeper integration of electrical components. There are multiple ongoing efforts to develop digital hydraulic pumps and motors. One notable example is the check valve type digital displacement pump (DDP) developed by Artemis Intelligent Power Ltd (Artemis). In collaboration with Danfoss Power Solutions, this machine is now being offered as a commercial product and boasts a faster response rate and a higher efficiency than the traditional swash-plate type machines. Various designs for digital hydraulic pumps are extant in patents and academic publications. In the present work, "digital displacement" refers to technology developed by Artemis. The distinction seems appropriate since Artemis coined the term and holds trademark rights to its use for commercial purposes.

The efficiency characteristics of swash-plate pumps are well known. Methods for measuring and calculating efficiency are documented in textbooks and international standards [3, 4]. The conventional definitions of mechanical and volumetric efficiency are not appropriate for digital displacement pumps. As has been shown in previous works [5..7], conventional formulas underestimate the volumetric efficiency of a digital displacement pump and overestimate its mechanical efficiency. By the conventional method, DDP's mechanical efficiency is greater than one, which is clearly meaningless. Consequently, DDP energy characteristics are typically given in terms of total efficiency because it is readily calculated from measurements of power input and output.

Nevertheless, traditions are persistent. Fluid power engineers are accustomed to thinking of steady-state pump characteristics in terms of volumetric and mechanical efficiency. To facilitate the acceptance of a new technology, it is helpful to speak in familiar terms. To that end, an accurate calculation of partial efficiencies for a DDP requires a new definition of ideal pump displacement. Displacement calculations suitable for DDP were presented in recent publications [5..7]. These methods are much better than conventional concepts. Upon further review, additional opportunities for improvement have been identified. The aim of the present paper is to provide a more accurate definition of DDP partial efficiencies based on a derivation of ideal torque and flow rate.

Literature Review

The basic digital displacement concept is individually controlling pistons within a piston pump or motor, thereby dividing the flow output into discrete steps or pulses. The piston control valves also have discrete on-off states. Research in this direction was first published in 1984 by Salter and Rea [8] from the University of Edinburgh wherein the conceptual design of a digital displacement pump was proposed. Continued research resulted in patent applications in 1989 and 1990 and a PhD thesis by Rampen in 1992 with measurements from a working prototype [9..11]. The researchers in Edinburgh formed Artemis as a technology start-up company in 1994 and continued to develop the technology. Partnerships with established manufacturers including Danfoss resulted in many more patents and publications in the last 25 years. Digital displacement pump/motor technology has been successfully demonstrated in applications including industrial machinery, wind power, rail, automotive vehicles and mobile off-highway machinery.

Academic interest in digital hydraulic pumps has greatly increased in the last decade. Research is ongoing at multiple universities in Europe and North America, which was partly summarized in a review paper by Linjama [12]. Researchers are particularly active at a trio of European universities located in Tampere, Finland; Aalborg, Denmark; and Linz, Austria. Ten annual workshops on digital fluid power have been hosted by these three universities. The University of Minnesota is another frequent contributor. Other academic conferences about fluid power technology regularly include papers about digital pumps and motors.

Digital pump/motor concepts can be divided into two basic groups. The first group is based on high-speed switching, similar to pulse-width modulation in power electronics [13..16]. The second type uses digital control of individual pistons. Digital displacement is part of the latter category. Various designs have been proposed for digital pumps, motors, pump-motors and hydraulic transformers with independent piston control [17..19]. While the literature illustrates a growing trend toward digital hydraulics and check valve type pumps [20..24], their theoretical characteristics are worthy of further study.

Standard methods exist for experimentally determining the volumetric displacement of positive displacement hydraulic pumps [3, 4]. Other methods can be found in textbooks and scholarly papers. According to these methods, a pump's volumetric displacement is defined to be constant with respect to pressure. Fluid compressibility effects are then considered to be volumetric losses. Such methods are well-suited for common hydraulic pumps and motors, including axial piston and gear type machines. Digital displacement pumps use a radial piston configuration with check valves for flow control. Experience with check valve pumps has shown that the apparent displacement of the pump shrinks as the discharge pressure increases. That is, the input torque and the output flow rate at high pressure are both lower than would be expected if displacement was constant. This phenomenon of pump displacement shrinkage is known to pump designers to be related to fluid compressibility.

Caldwell was the first author to publish a mathematical model of volumetric displacement for check valve pumps which explicitly considers displacement shrinkage due to fluid compressibility [25]. He noted the need to separate the effects of leakage and compressibility, both of which reduce the output flow. He then created a semi-empirical loss model based on steady-state pump measurements. Once torque and flow losses are known, equations for the ideal volumetric displacement as well as volumetric and mechanical efficiency are readily derived. The authors of the current paper also published expressions for ideal displacement volume from which partial efficiencies can be calculated [5]. Their approach considered a theoretical analysis of an ideal DDP, rather than an empirical efficiency model. In either case, all the authors assumed that displacement shrinkage affects input and output power equally. It will be shown in the current work that the shrinkage at the input and output are slightly different. There were other simplifying assumptions in the previous work--Caldwell's formulas were limited to a full-strokes flow algorithm, and Manring and Williamson's method had a small inaccuracy due to a linearization of the exponential function in the derivation. These concerns are addressed in the present work.

PUMP DESCRIPTION

An operational schematic of the digital displacement pump is shown in Figure 1. The left side of this figure illustrates the pump as a single pumping unit represented by a piston that reciprocates following an eccentric cam. It is important to note that an actual pump comprises multiple pumping units depending upon the design choice of the manufacturer. Also, in the analysis which follows, the mechanism for displacing the piston is not limited to the piston-cam device shown. The mechanism could be a rolling cam follower, slider-crank device or any reciprocating mechanism that produces a cyclic change

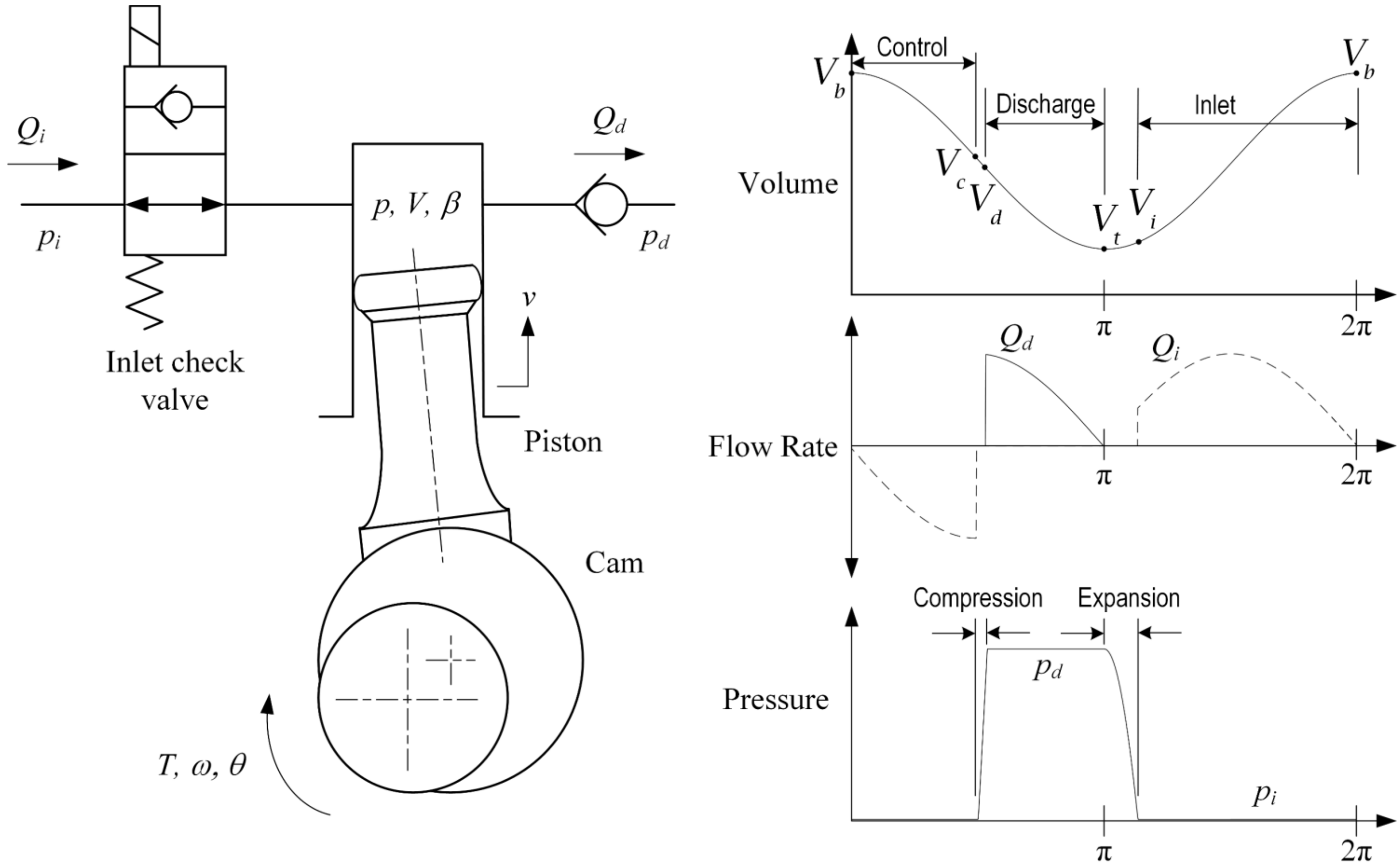

Figure 1: Geometry and operation of a single piston in the digital displacement pump

in volume for the piston chamber. In Figure 1, the pressure within the piston chamber is shown by the symbol p and the instantaneous volume of fluid within this chamber is given by V. The symbol β is used to represent the fluid bulk modulus of elasticity which describes the compressibility of the fluid.

In Figure 1 the left side of the piston chamber is shown to use an actively controlled check valve and a passive check valve in parallel for connecting or separating the piston chamber from the intake pressure p_i. In the de-energized state, the inlet check valve connects the piston chamber to the inlet in such a way as to facilitate the inlet flow Q_i while keeping the piston pressure p essentially equal to the inlet pressure p_i. The inlet check valve provides very little resistance to fluid flow in the open position. When the solenoid is energized, flow into the piston chamber is controlled by the check valve. As long as the piston pressure p remains slightly lower than the inlet pressure p_i the inlet flow Q_i is facilitated by the check valve while keeping the piston pressure p essentially equal to the inlet pressure p_i. On the right side of the piston chamber, a passive check valve is used for connecting or separating the piston chamber from the discharge pressure p_d. As long as the piston pressure p remains slightly higher than the discharge pressure p_d the discharge flow Q_d is facilitated by the check valve while keeping the piston pressure p essentially equal to the discharge pressure p_d. The proper switching of the inlet check valve is used to adjust the amount of fluid that is pumped across the discharge check valve, thereby adjusting the commanded volumetric displacement of the machine.

As previously noted, Figure 1 shows a shaft with an eccentric cam that actuates a piston. The piston moves mostly in the vertical direction and wobbles horizontally as the shaft rotates. The rotational power input is shown by the torque T which is applied to the cam while the cam rotates at an angular velocity ω. This rotational input displaces the pressurized piston in the vertical direction while producing an instantaneous velocity shown in Figure 1 by the symbol v. The charts on the right side of Figure 1 show the stages of pump operation, beginning with the piston at bottom dead center. At the starting position, the cylinder volume is at its maximum, equal to V_b.

Stage 1 – Control. The inlet check valve is in the open position, connecting the piston chamber to the inlet side of the pump. In this condition, the piston pressure is essentially equal to the inlet pressure. Valve flow rates and piston pressure are shown in the middle and bottom charts on the right side of Figure 1. As the piston moves up, the displaced volume of fluid in the piston chamber is pushed *out* of the inlet side of the pump, creating a negative flow rate for Q_i. The important thing to notice is that the displaced fluid does *not* pass into the discharge port since the piston pressure is much lower than the discharge pressure and the discharge check valve is closed. In this stage, the pump's average discharge flow is reduced by purposely diverting fluid to the inlet.

Stage 2 – Compression. At the beginning of the second stage of pump operation, the inlet check valve has changed to the check valve position and the check valve has just closed, disconnecting the piston chamber from the inlet side of the pump. At this instant, the volume of fluid in the piston chamber is given by V_c as shown by the top chart in Fig. 1. During this stage the piston continues to advance into the piston chamber while increasing the piston pressure above the inlet pressure, but not having yet raised the pressure to the level of the discharge pressure. In other words, this stage of pump operation is carried out while both check valves are closed and the fluid within the piston chamber is being compressed. When the pressure in the piston chamber reaches the discharge pressure of the pump, the instantaneous fluid volume in the piston chamber is given by V_d.

Stage 3 – Discharge. At the beginning of the third stage of pump operation, the discharge check valve has just opened to connect the piston chamber to the discharge side of the pump. Again, the volume of fluid in the piston chamber at this point is given by V_d. During this stage, the piston continues to advance into the piston chamber thus pushing fluid through the discharge check valve of the pump. Obviously, this is the operating stage where the displacement of the pump contributes to the discharge flow. Since the piston volume is still diminishing during this stage, the discharge flow rate is positive.

Stage 4 – Expansion. At the beginning of the fourth stage of pump operation the piston is located at top dead center where the fluid volume in the piston chamber is minimum and given by the symbol V_t. At this position, the angular displacement of the pump shaft is shown in Figure 1 when θ equals π. As the shaft continues to rotate and the piston begins to retract from the piston chamber, the volume in the piston chamber expands and the fluid pressure begins to drop. Since the fluid pressure remains between the discharge and inlet pressures of the pump, the check valves on both sides of the pump are closed and no fluid exits or enters the machine. This stage of pump operation reduces the amount of fluid drawn into the pump from the inlet side. Furthermore, the relaxation of the pressurized fluid during this stage is used to *assist* the torque on the input shaft in the direction of pump rotation, thus recapturing some of the energy that was used to compress the fluid during the compression stage of pump operation.

Stage 5 – Inlet. At the beginning of this final stage of pump operation, the expansion stage has been completed and the fluid pressure within the piston chamber is now equal to the inlet pressure of the pump. At this point, the fluid volume in the piston chamber is given by V_i and the inlet check valve opens to facilitate flow into the piston chamber. Under these conditions a positive inlet flow rate Q_i enters the pump. See the middle chart on the right side of Figure 1. During this stage the piston chamber fills with new fluid until the piston reaches bottom dead center where the cyclic repetition of the five stages of pump operation begins again.

FLOW ALGORITHM

Before beginning the analysis, it may be instructive to say a few words about control algorithms. There are basically two ways to achieve a variable flow rate from the DDP: partial piston strokes and full piston strokes. The partial stroke method was described previously. The on-off control valve is intentionally held in the closed-switch position during part of the piston's upstroke. Only a portion of the piston's swept volume ΔV is discharged through the outlet check valve. With the partial stroke algorithm, the commanded displacement is represented by x, where $x = (V_c - V_t) / \Delta V$ and is continuously variable between 0 and 1. For a pump with multiple pistons, a simple partial stroke algorithm assumes that the commanded volume fraction x is the same for all pistons.

With a full stroke algorithm, the entire swept volume of each piston is pumped either to the inlet or to the outlet. Flow rate is modulated by selectively enabling and disabling pistons as the shaft rotates. The full stroke algorithm only pressurizes the minimum number of pistons N required to satisfy the commanded volumetric displacement. Obviously, some displacement fractions can be achieved with integer values of N. For example, if the pump has 12 pistons, $N = 9$ gives a displacement fraction of 9/12 = 0.75. Nine pistons are active and three are idle during every revolution. N can also have non-integer values, such as $N = 9.5$. Then the displacement fraction is 9.5/12 = 19/24 = 0.792. Nine pistons are active during the first revolution, and 10 pistons are active during the second revolution, always with full strokes ($x = 1$). The desired displacement fraction is satisfied as an average over time. The full stroke algorithm is analogous to an analog-to-digital converter. An analog value (desired displacement fraction) is converted to a sequence of ones and zeros (on and off pistons). Any analog value can be converted to a sufficiently long binary sequence. In practice, a reasonable compromise can be found between the resolution of the desired flow rate and the maximum length of the piston control sequence.

Beyond these two basic flow algorithms, a desired displacement fraction could be satisfied with many possible combinations of full and partial piston strokes. A more detailed discussion of flow algorithms is beyond the scope of the present work. For the problem at hand, the important point is that a general description of the DDP's ideal displacement volume must consider both flow algorithms.

IDEAL TORQUE AND FLOW RATE

The ideal average torque can be derived by integrating $p \cdot dV$ over one shaft revolution. The complete analytical derivation can be found in a previous work [7]. The average torque over one revolution, in terms of piston volumes, is given by

$$\mathrm{T_{ideal}} = \frac{\mathrm{N}}{2\pi}\beta(\mathrm{V_c} - \mathrm{V_i} - \mathrm{V_d} + \mathrm{V_t}) \quad (1)$$

This is the ideal pump torque, including the effect of fluid compressibility. Equivalently, the average torque may be written in terms of fluid pressures and bulk modulus.

$$T_{\text{ideal}} = \frac{N}{2\pi}\beta\left(x\Delta V + V_t - V_t e^{\frac{p_d - p_i}{\beta}} - V_c e^{\frac{p_i - p_d}{\beta}} + V_t\right) \quad (2)$$

To write this equation in a more familiar form, we will substitute a second-order Taylor series expansion for the exponential function, $e^y \cong 1 + y + \frac{y^2}{2}$. With $\Delta p = p_d - p_i$ and a bit of rearranging, we obtain a more compact expression for the ideal torque.

$$T_{\text{ideal}} \cong \frac{N\Delta V\Delta p}{2\pi}\left[x - \left(\frac{x}{2} + \frac{V_t}{\Delta V}\right)\frac{\Delta p}{\beta}\right] \quad (3)$$

The reader may notice that equation (3) contains both N and x, indicating that it is valid for both partial and full stroke flow algorithms. If we neglect fluid compressibility (let $\beta=\infty$), equation (3) reduces to a conventional expression for pump torque.

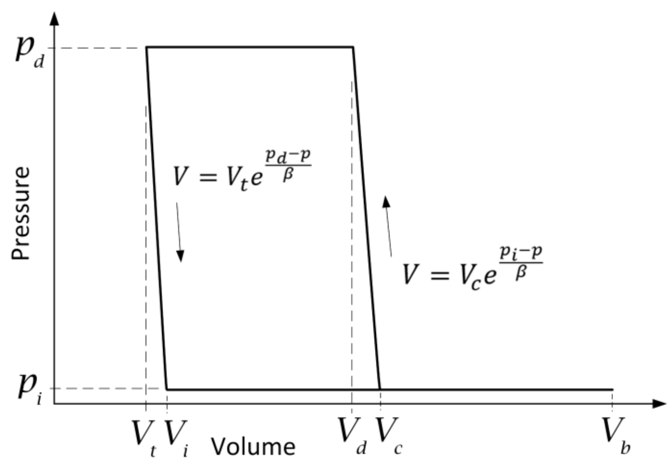

Figure 2: Pressure-volume diagram for a single piston, x = 0.5

The ideal torque can also be derived graphically. Pressure-volume diagrams are commonly used in the analysis of internal combustion engines, refrigeration equipment and other cyclical machines. Figure 2 depicts the pressure and volume of a single piston over one shaft revolution. Torque is proportional to the area enclosed by the curves in the pV plane. Let's approximate the area with rectangles and triangles and sum the torques, noting that the expansion torque is negative. The average torque for a single piston is then given by

$$\begin{aligned} T &\cong \Delta p\left((V_d - V_t) + \frac{V_c - V_d}{2} - \frac{V_i - V_t}{2}\right) \\ &\cong \frac{\Delta p}{2}(V_d - V_t + V_c - V_i) \end{aligned} \quad (4)$$

Assuming triangular shapes implies a linear relationship between pressure and volume. That is,

$$\begin{aligned} V_d &\cong V_c\left(1 - \frac{\Delta p}{\beta}\right) \\ V_i &\cong V_t\left(1 + \frac{\Delta p}{\beta}\right) \end{aligned} \quad (5)$$

We can also use the relationship $V_c = x \cdot \Delta V + V_t$ as defined previously. Substituting into (4) and simplifying, the single-piston torque is

$$T \cong \Delta V\Delta p\left[x - \left(\frac{x}{2} + \frac{V_t}{\Delta V}\right)\frac{\Delta p}{\beta}\right] \quad (6)$$

For the torque per revolution with multiple pistons, multiply by $N/2\pi$ to obtain the same equation as (3).

$$T_{\text{ideal}} \cong \frac{N\Delta V\Delta p}{2\pi}\left[x - \left(\frac{x}{2} + \frac{V_t}{\Delta V}\right)\frac{\Delta p}{\beta}\right] \quad (7)$$

Similarly, the ideal discharge flow rate Q_{ideal} can be found by integrating the piston volume over one shaft revolution.

$$Q_{\text{ideal}} = -\frac{N}{2\pi}\omega\int_{V_d}^{V_t} dV = \frac{N}{2\pi}\omega(V_d - V_t) \quad (8)$$

In terms of pressure, the discharge flow rate is

$$Q_{\text{ideal}} = \frac{N\omega}{2\pi}\left(V_c e^{\frac{p_i - p_d}{\beta}} - V_t\right) \quad (9)$$

There is a graphical analogy here, too. The discharge flow rate is proportional to the length of the top horizontal line in Figure 2, and the inlet flow rate is proportional to the length of bottom line. As before, substitute a series approximation for the exponential function $e^y \cong 1 + y + \frac{y^2}{2}$ and rearrange the terms to be similar to (7).

$$Q_{\text{ideal}} \cong \frac{N\Delta V\omega}{2\pi}\left[x - \left(x + \frac{V_t}{\Delta V}\right)\left(\frac{\Delta p}{\beta} - \frac{\Delta p^2}{2\beta^2}\right)\right] \quad (10)$$

Let's examine equations (7) and (10). For convenience, define dimensionless displacement factors equal to the terms in square brackets. d_T is related to the input power and d_Q is related to the output power.

$$d_T = x - \left(\frac{x}{2} + \frac{V_t}{\Delta V}\right)\frac{\Delta p}{\beta} \quad (11)$$

$$d_Q = x - \left(x + \frac{V_t}{\Delta V}\right)\left(\frac{\Delta p}{\beta} - \frac{\Delta p^2}{2\beta^2}\right)$$

These equations show how the apparent pump displacement changes with pressure. It is obvious from (11) that displacement shrinks with increasing discharge pressure. It is also clear that $d_T \neq d_Q$. That is, the displacement shrinkage as seen from the input shaft is not equal to the displacement shrinkage seen in the discharge flow rate. Why does pressure affect apparent displacement differently at the input and output? On the input side, torque is affected by fluid compression *and* expansion. As the discharge pressure level increases, more torque is required to compress the fluid. Part of the energy of the compressed fluid is returned to the shaft during the expansion phase. Averaging over a complete cycle, the positive compression torque is partially offset by the negative expansion torque. The compression torque is greater than the expansion torque because $V_c > V_t$. On the output side of the pump, the discharge flow rate is only affected by fluid compression. In equations (9) and (10), the discharge flow rate shrinks due to the compression of the fluid before the outlet check valve opens. The outlet flow is unaffected by the expansion stage because the outlet check valve is closed at that time. Therefore, displacement shrinkage has a greater effect at the output than the input. Figure 3 illustrates this effect with typical values for the constants.

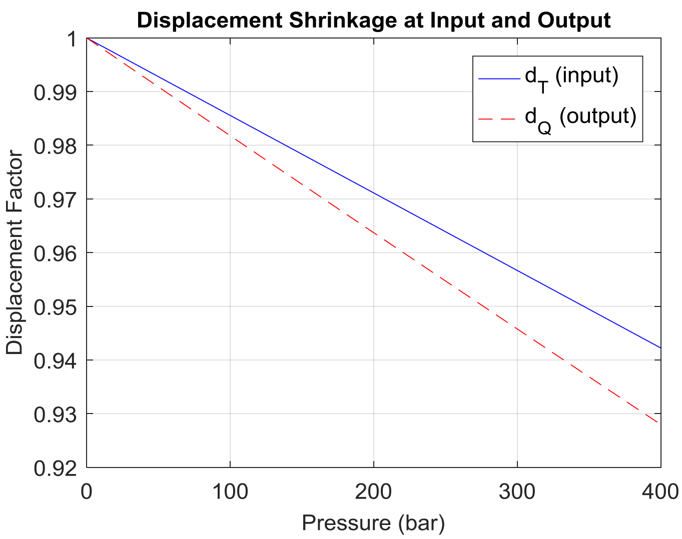

Figure 3: Comparison of displacement factors with $x = 1$, β =1.3 GPa, $V_t / \Delta V = 1.4$

Although β is assumed to be constant in Figure 3, it is not a necessary assumption in the analysis. Considering the variation of bulk modulus with fluid pressure and temperature is encouraged if sufficient fluid property data are available.

EFFICIENCY

The discussion of input versus output shrinkage begs a question about efficiency and conservation of energy. Given that the DDP's apparent displacement shrinks with pressure, and given that the output shrinks more than the input, what about the difference between input and output? Should not the efficiency of an ideal pump be unity regardless of pressure? The answer is no, because the conventional definition of pump efficiency disregards the energy required to compress the fluid.

Consider the conventional definitions of pump efficiency, where η_m is hydromechanical efficiency, η_v is volumetric efficiency and η is total efficiency.

$$\eta_m = \frac{\Delta p \hat{V}}{2\pi T_{\text{actual}}} = \frac{\text{ideal torque}}{\text{actual torque}} \tag{12}$$

$$\eta_v = \frac{2\pi Q_{\text{actual}}}{\hat{V}\omega} = \frac{\text{actual flow rate}}{\text{ideal flow rate}} \tag{13}$$

$$\eta = \eta_m \eta_v = \frac{p_d Q_{\text{actual}}}{\omega T_{\text{actual}}} = \frac{\text{output power}}{\text{input power}} \tag{14}$$

In these conventional definitions, $\hat{V}$ is defined at zero pressure where the efficiency of an ideal pump is one. For any nonzero pressure, $Q_{\text{actual}} < \hat{V}\omega$ due to fluid compression. The volumetric efficiency must then be less than one, meaning that the flow reduction due to compression is counted as a volumetric loss. The total efficiency is also less than one, meaning that the energy required to compress the fluid is defined to be a power loss. This is true for any positive displacement pump.

Since we have just derived equations for the ideal torque and flow rate of a digital displacement pump, one might assume that its volumetric efficiency would be defined as a ratio of ideal and measured flow rates, as in (13). Surprisingly, this approach is incorrect. The product of the partial efficiencies would not be equal to the total efficiency. It can be shown that, for DDP,

$$\frac{T_{\text{ideal}}}{T_{\text{actual}}} \cdot \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} \neq \frac{\Delta p Q_{\text{actual}}}{\omega T_{\text{actual}}} \tag{15}$$

The conventional approach fails because the definition of total efficiency counts fluid compression as a power loss. If we explicitly consider compressibility, as in (9) and (10), then the efficiency of an ideal pump is always unity, and the volumetric efficiency of any real pump is overestimated. Instead, we must define volumetric efficiency such that the product of volumetric and mechanical efficiencies is equal to the total efficiency. Using equations (7), (10) and (11), we can write an expression for total efficiency in terms of ideal torque and flow rate.

$$\frac{T_{\text{ideal}}}{T_{\text{actual}}} \cdot \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} \cdot \frac{d_Q}{d_T} = \frac{\Delta p Q_{\text{actual}}}{\omega T_{\text{actual}}} = \eta \tag{16}$$

The definitions of partial efficiency for DDP are then straightforward.

$$\eta_{m,\text{DDP}} = \frac{T_{\text{ideal}}}{T_{\text{actual}}} = \frac{N \Delta V \Delta p d_T}{2\pi T_{\text{actual}}} \tag{17}$$

$$\eta_{v,\text{DDP}} = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} \cdot \frac{d_Q}{d_T} = \frac{2\pi Q_{\text{actual}}}{N \Delta V \omega d_T} \tag{18}$$

As an illustration, Figure 4 shows the total efficiency of an ideal DDP as a function of outlet pressure. The efficiency is less than unity for nonzero pressure, as expected. The red dashed line was calculated with equations (2) and (9). The blue line was calculated with equation (11). Displacement factors d_Q and d_T are defined with a Taylor series approximation, as discussed previously. Figure 4 demonstrates the accuracy of the series approximations implicit in equations (11). Using typical values for fluid compressibility and piston dead volume, the difference between total efficiency based on the power ratio and d_Q / d_T is 0.00017 at 400 bar. Furthermore, Figure 4 demonstrates the validity of equation (16). For an ideal DDP, the ratio of output power to input power is equal to the ratio of output shrinkage to input shrinkage. If displacement shrinkage is assumed to affect input and output channels equally, as was done in previous work, the associated error in partial efficiencies is equal to d_Q/d_T. For typical pump and fluid parameters, the maximum error is < 2%.

The reader may note that displacement shrinkage (see Figure 3) and maximum ideal efficiency (see Figure 4) are separate effects, both related to fluid compressibility. DDP displacement decreases with pressure due to the operation of the check valves in the same way as other check-valve or check-ball type pumps. The decrease in maximum total efficiency with pressure is due to the compression of the fluid discharged from the outlet, which is true for all positive displacement pumps. The magnitude of the compressibility effects depends on the pump design.

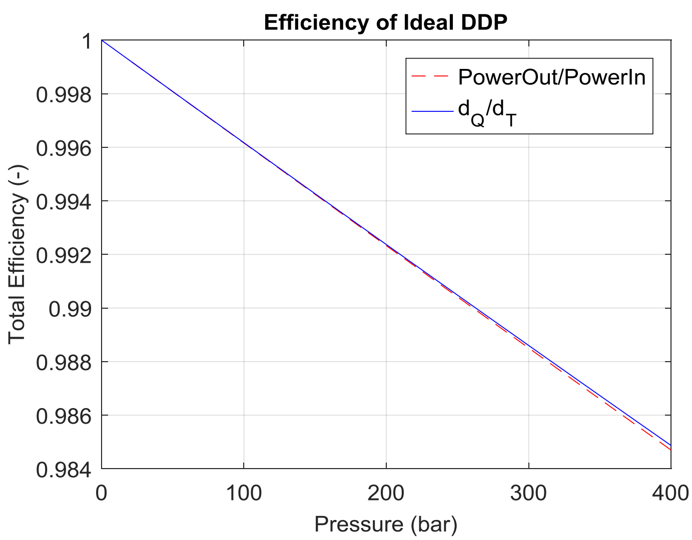

Figure 4: Total efficiency of an ideal DDP with x = 1, β=1.3 GPa, Vt/ΔV = 1.4

DERIVED DISPLACEMENT VOLUME

The definitions of volumetric and mechanical efficiency require the pump displacement volume to be known. ISO 8426 says that the displacement volume should be determined experimentally [3]. The standard test procedure is to measure the pump's outlet flow rate with increasing pressure at constant speed, temperature and commanded displacement. The derived displacement, which we denote by $\hat{V}$, is found by calculating $Q_{\text{actual}} / \omega$ at each measured pressure, fitting a least-squares line to the points and extrapolating to $p_d = 0$. The value of $\hat{V}$ is defined to be the zero-pressure intercept of the line as shown in Figure 5.

Alternatively, according to SAE J745, displacement volume is measured at constant pressure and variable speed [4]. Discharge pressure is set to 5% of the pump's rated maximum, and a number of speeds are selected over the rated speed range. The derived displacement is found by plotting measured discharge flow rate versus speed and calculating the slope of a line fitted to the measured points.

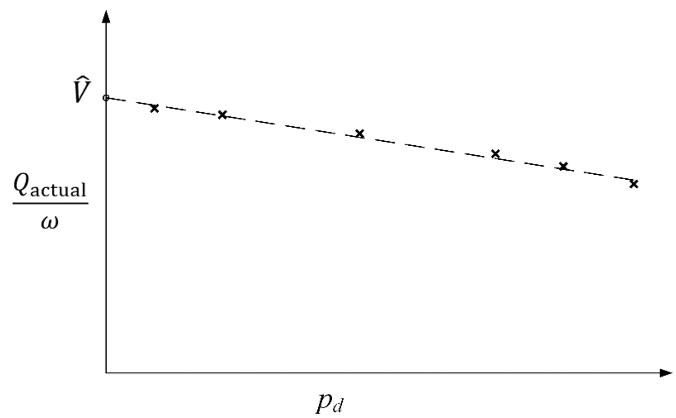

Figure 5: Zero-pressure intercept method for derived displacement volume

Another popular method is a two-step process that combines the two methods mentioned previously [26]. According to Toet, derived displacement volume is found by first calculating the slope $\partial Q / \partial \omega$ at various pressures and second extrapolating the slope or displacement values to zero pressure. By Toet's method, $\hat{V}$ = 94.5 at 50° C and 95.6 at 80° C for the tested DDP. For the purposes of the present work, all of these methods for determining $\hat{V}$ are acceptable, since $\hat{V}$ is defined to be constant with respect to pressure.

What is the derived displacement volume of a digital displacement pump? It has already been shown that $\hat{V}$ alone is not a useful quantity for DDP because of displacement shrinkage. The efficiency equations in the previous section are given in terms of the pump's geometric volumes ΔV and V_t and control parameters N and x. All of these parameters affect the outlet flow rate. The radial piston pump design ensures that the swept volume ΔV and the dead volume V_t are constant. The number of pistons in use N is known exactly due to the digital flow algorithm. The remaining variable is x, the commanded partial stroke displacement. Variation in the measured displacement is primarily due to valve timing. To produce the desired displacement exactly, the inlet valve must close at precisely the right moment. Due to manufacturing tolerances and the effect of operating conditions (such as temperature), there is some variation in valve response time. Therefore, it makes sense to account for these variations with x, such as when calculating partial efficiencies with equations (17) and (18).

Consider Figure 6, which shows experimental results from a DDP efficiency test. From 500 through 2500 rpm, the variation in $\hat{V}$ corresponds to x = 0.96 to 0.98. Above the rated speed of

2500 rpm, where the flow output decreases sharply, a different phenomenon is at work. The reduction in output flow at high speed is due to inlet cavitation rather than control valve timing. In such a condition, it seems more appropriate to account for the reduction in output by reducing ΔV rather than x.

EXPERIMENTAL RESULTS

The efficiency characteristics of a digital displacement pump were evaluated experimentally. Pump efficiency tests were conducted at the Danfoss facility in Ames, Iowa, USA according to internal and international measurement standards. The pump on test was a 96 cc/rev radial piston unit supplied by Artemis. A detailed description of the experiment was published previously [5]. The reader should keep in mind that the pump was a prototype unit. The results shown here are not intended to be representative of pumps in serial production.

Figure 6 shows the experimentally derived maximum displacement according to the zero-pressure intercept method. The data shown in Figure 7 through Figure 10 was measured at 1500 rpm, 50 degrees Celsius and 100% commanded displacement. For calculating partial efficiency, we used x = 0.969 to agree with the derived displacement of 95.3 cc/rev at the given speed and temperature.

Shrinkage in measured torque and flow is visible in Figure 7. The ideal curves are between the actual flow and torque curves, as they should be. The actual flow is close to the ideal flow at low pressure, which also makes sense. The DDP does not have cross-port leakage between discharge and inlet volumes, so one would expect the volumetric losses to be very low at zero pressure.

Figure 8 shows the pump's total and partial efficiencies. As intended, the total efficiency as a product of partial efficiencies is equal to the ratio of output power to input power.

Calculating DDP's partial efficiencies without accounting for displacement shrinkage yields nonsensical results, as illustrated in Figure 9 and Figure 10. If displacement is taken to be independent of pressure, as prescribed by ISO 8426 and ISO 4409, the mechanical efficiency can be greater than one, and the volumetric efficiency can be less than the total efficiency. Accounting for the effects of fluid compressibility gives more reasonable results, as has been previously published [5]. The improved definitions of partial efficiency in the present work are up to 2% more accurate than the previous method.

The data shown in Figure 11 was also measured at 1500 rpm and 50 degrees Celsius, though with a 50% commanded displacement and the partial strokes flow algorithm.

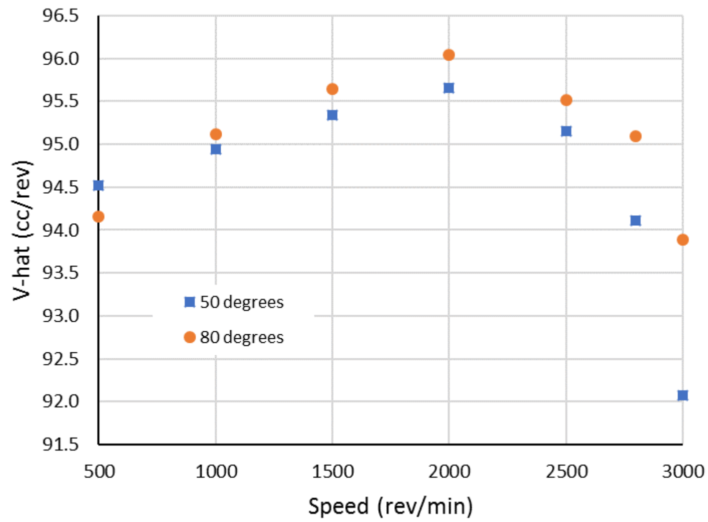

Figure 6: DDP derived displacement volume per ISO 8426 at maximum commanded displacement

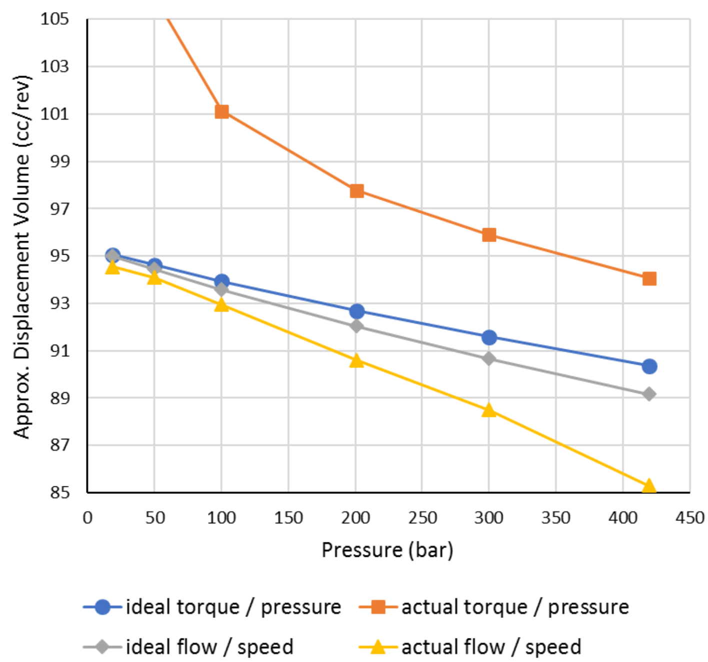

Figure 7: Ideal and actual displacement shrinkage

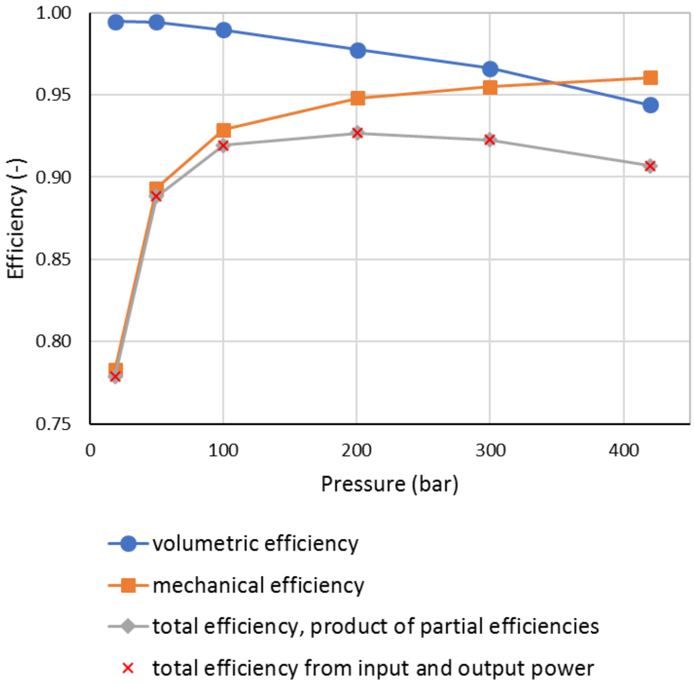

Figure 8: Partial and total pump efficiency at maximum displacement

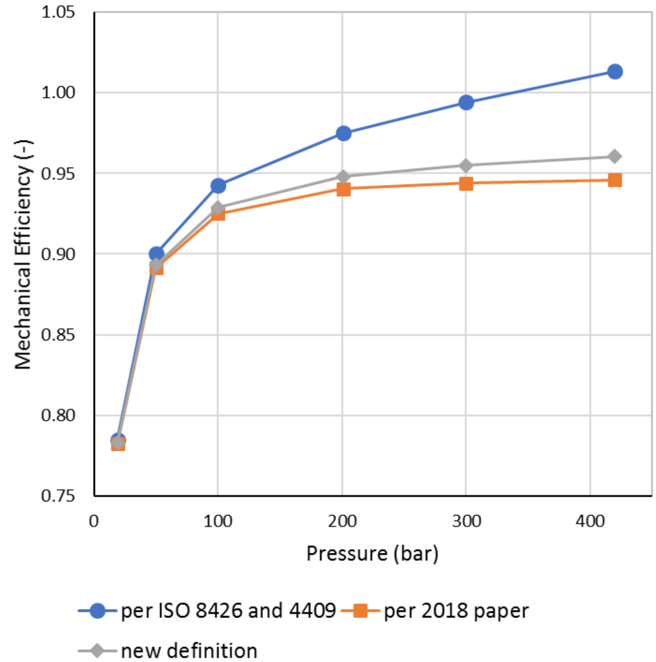

Figure 10: Comparison of DDP mechanical efficiency

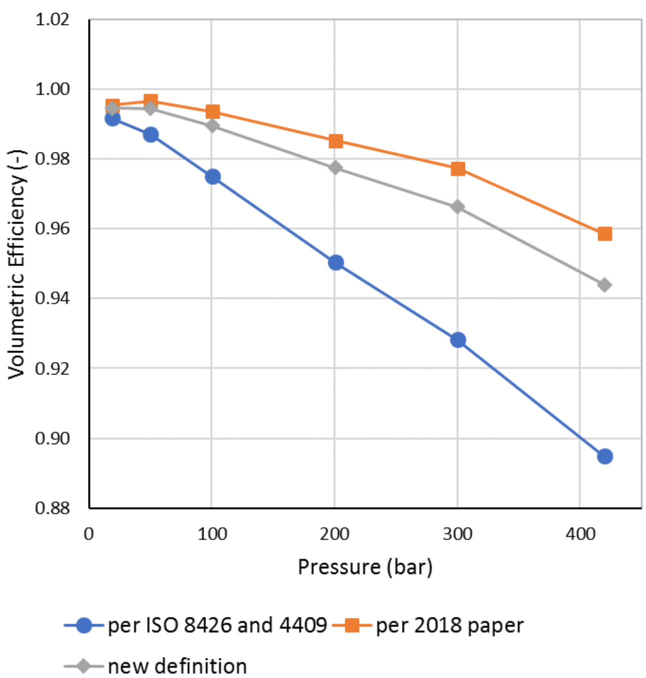

Figure 9: Comparison of DDP volumetric efficiency

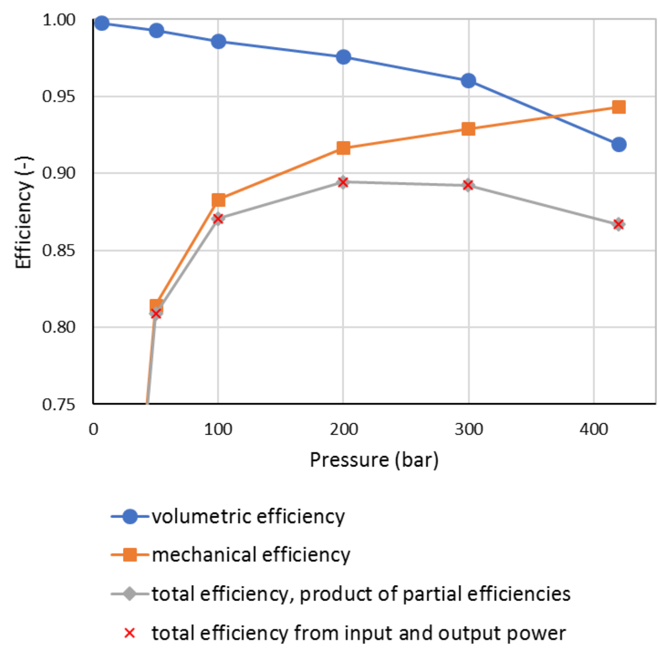

Figure 11: Partial and total pump efficiency at 50% commanded displacement with a partial strokes flow algorithm.

CONCLUSIONS

The following conclusions are supported by the analysis and results presented in this paper:

1. Fluid compressibility effects within a digital displacement pump (DDP) reduce the apparent volumetric displacement of the machine. This displacement shrinkage affects the output (flow rate) slightly more than the input (torque) due to the difference in compressed fluid volume at the beginning and end of each piston stroke and the operation of the check valves. Following the conventional definition of total efficiency, the difference between input and output shrinkage is counted as a power loss.
2. Without accounting for the apparent pump shrinkage, computations of the volumetric and mechanical efficiencies are unrealistic. By conventional formulas, DDP's mechanical efficiency often exceeds unity and its volumetric efficiency is less than the total efficiency. The correct partial efficiencies may be computed by accounting for the pressure-dependent displacement shrinkage. Experimental data indicate that the correct results differ from the uncorrected results by as much as 5%.
3. Shrinkage at the pump input and output were considered to be equal in previous publications. The new results improve the accuracy of computed volumetric and mechanical efficiencies by up to 2%.
4. Partial pump efficiencies are typically calculated with an empirically derived displacement volume. A method is proposed for determining a DDP's derived displacement volume. The experimental procedure follows the ISO standard while the application to pump efficiency analysis uses parameters that are unique to DDP.

While seeking to take advantage of DDP's high efficiency and fast dynamic response, the conclusions of this research are useful for serving an industry that is accustomed to considering the volumetric and mechanical efficiencies of positive displacement pumps.

The equations in this paper are intended specifically for digital displacement pumps. Similar equations may be derived for digital displacement motors, which may be considered in a future publication. The concept of displacement shrinkage is applicable to check-valve type pumps in general, and may become more widely known as digital fluid power wins increasing research investment and commercial acceptance.

ACKNOWLEDGMENTS

The research reported in this paper was supported by Danfoss Power Solutions. The authors gratefully acknowledge the contributions of Prof. Luca Zarotti, including the inequality of input and output shrinkage and the graphical derivation of ideal pump torque.

NOMENCLATURE

dQ nondimensional output displacement factor
dT nondimensional input displacement factor
N number of pistons in use within the pump
p instantaneous fluid pressure within a single piston chamber
p_d discharge pressure
p_i inlet pressure
Q_{actual} measured discharge flow rate
Q_d volumetric flow rate through the discharge check valve for a single piston
Q_i volumetric flow rate through the inlet check valve for a single piston
Q_{ideal} average ideal flow rate exiting the pump at the discharge port
T torque required to drive a single piston
T_{actual} measured pump shaft torque
T_{ideal} average ideal torque at the pump shaft
V instantaneous volume of a single piston chamber
V_{hat} pump displacement volume derived by conventional methods
V_b volume of a single piston chamber at bottom dead center
V_c volume of a single piston chamber when it becomes connected to the pump inlet
V_d volume of a single piston chamber at the end of the compression stage
V_i volume of a single piston chamber at the end of the expansion stage
V_t volume of single piston chamber at when the piston is located at top dead center

v upward velocity of a single piston
x nondimensional commanded displacement of the pump using partial piston strokes

β fluid bulk-modulus of elasticity
ΔV swept volume of a single piston chamber
η overall pump efficiency
η_m mechanical pump efficiency
η_v volumetric pump efficiency
θ angular position of the pump shaft
ω angular velocity of the pump shaft

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